Adding Missing-Data-Relevant Variables to FIML-based Structural Equation Models

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Abstract
Conventional wisdom in missing data research dictates adding variables to the missing data model when those variables are predictive of (a) missingness, and (b) the variables containing missingness. However, it has recently been shown that adding variables that are correlated with variables containing missingness, whether or not they are related to missingness, can substantially improve estimation (bias and efficiency). Including large numbers of extra variables is straightforward for researchers who use multiple imputation. However, what is the researcher to do if FIML/SEM procedures are the analysis of choice? This article suggests two models for SEM analysis with missing data, and presents simulation results to show that both models provide estimation that is clearly as good as analysis with the EM algorithm.
In the missing data literature (e.g., Little & Rubin, 1987), data are often referred to as being made up of two parts: \( Y_{\text{obs}} \) (the data that are observed) and \( Y_{\text{mis}} \) (the data that are missing). The definition of the term "missing at random" (MAR) in this context is that the "cause" of missingness may be related to \( Y_{\text{obs}} \), but not to \( Y_{\text{mis}} \). It has long been conventional wisdom among missing data theorists and practitioners that including these MAR causes of missingness in the missing data model will improve estimation, in particular, by reducing estimation bias.

Recently, Collins, Schafer, and Kam (2001) have shown that including these MAR causes in the missing model does help, but only when (a) there is a substantial amount of missing data (e.g., 50% or more), and (b) when the variable causing missingness is substantially correlated with the variable containing missingness (e.g., when \( r > .40 \)), and (c) when the variable causing the missingness is substantially correlated with missingness (e.g., when \( r > .40 \)). In addition, Collins et al. (2001) have shown that linear MAR conditions are not the only ones with which researchers must be concerned. They have shown that two non-linear missing data conditions, which are fully MAR, have different effects on estimation bias than does linear MAR missingness.

Perhaps the most intriguing finding of Collins et al. (2001) is that it can often be very helpful to estimation, both in terms of bias and efficiency, to include variables that are NOT the cause of missingness, but which nonetheless are highly correlated with the variable containing missingness. The fact that such variables are quite common makes this all the more important.

Because of the value for reducing bias and increasing efficiency, Collins et al. (2001) recommend that researchers should routinely include variables in the missing data model if they have high correlations with the variables containing missingness, whether or not they are part of the cause of missingness. Including these extra variables is straightforward for data-based procedures such multiple imputation (MI) or the EM algorithm for covariance matrices. One simply adds the extra variables to the analysis. In fact, Collins et al. (2001) have also shown that including extra variables into the MI model is done with little cost, such as estimation problems due to overfitting.

On the other hand, what is the researcher to do if one of the full-information maximum likelihood (FIML) procedures is the analysis of choice? FIML procedures currently in wide usage include LTA (Collins, Hyatt, & Graham, 2000; Hyatt & Collins, 2000), and structural equation modeling (SEM) programs Amos (Arbuckle & Wothke, 1999), and Mx (Neale, Boker, Xie, & Maes, 1999). The problem is this. How does the researcher include these extra variables in these FIML models without compromising the model of substantive interest?

In this article, I describe two general models for use with SEM programs. Both models allow the researcher to include variables that deal with the missing data aspects of the data, without affecting the substantive aspects of the model. In addition, I illustrate the value of these model with four brief simulations. I illustrate these points with the SEM program Amos (Arbuckle & Wothke, 1999), but the same models are readily applicable to other FIML/SEM (e.g., Mx; Neale et al., 1999) and similar programs (At present the kind of solution presented here is not available with LTA).

**Method**

**Specifying the Models**

Before describing the simulations, I first describe the manifest-, and latent-variable models to be used in this study. Model 1 is the model of substantive interest. It is a simple regression model with one predictor variable (X), and one dependent variable (Y). The manifest
variable version of this model is shown in Figure 1. The latent variable version is shown in
Figure 2. This model is referred to as the "EM Model" in the simulations. For all the
simulations, the EM analysis will always include the cause of missingness (a variable named
"RX"), but the SEM model, which is based on the EM covariance matrix, will be as shown in
Figures 1 and 2. It is assumed that this would be the model of choice if there were no missing
data.

Model 2, the "Extra DV Model", which was suggested by Graham, Hofer, Donaldson,
MacKinnon, & Schafer (1997), incorporates the RX variable into the model as an extra
dependent variable. RX is predicted by X, and the residual for RX is specified to be correlated
with the residual of the dependent variable, Y. The manifest variable version of Model 2 appears
in Figure 3. The latent-variable version of Model 2 appears in Figure 4.

Model 3 is referred to as the "Spider" model (see the latent variable version of this model
to see why it is so named). The extra variable, RX, is also incorporated into this model. In this
case, RX is allowed to be correlated with every manifest variable in the model. For completely
exogenous manifest variables, RX is correlated with the variable itself. For manifest variables
that are predicted, e.g., manifest variables that are indicators of a latent variable, RX is specified
to be correlated with the residual for that variable. The manifest variable version of Model 3
appears in Figure 5. The latent variable version appears in Figure 6.

Analysis with No Missing Data

Before turning to the performance of these models in the missing data case, their
performance is addressed when there are no missing data. It is important to see how these
models behave even with no missing data, so that their behavior with missing data can be
evaluated properly.

I reiterate that with Model 1, all the data, including the extra variable, RX, are submitted
to analysis with the EM algorithm, which produces a maximum-likelihood covariance matrix and
vector of means. This covariance matrix is then submitted to SEM analysis (e.g., with Amos).
The model tested is that shown in Figures 1 and 2, but the data have already been conditioned on
the extra variable, RX.

With the manifest variable models, the three models (Models 1, 2, and 3) yield identical
parameter estimates for the b-weight of X predicting Y. In fact, for the manifest variable models,
these three models yield identical parameter estimates whether there are missing data or not.

In the latent variable case, with no missing data, the EM and "Spider" models yield
identical parameter estimates for the regression weight of X predicting Y. However, for the
"Extra DV" model, the b-weight is very slightly different in the case of no missing data.

Dependent Variables for the Simulations

This study focuses on estimation bias and efficiency. The dependent variables to be used
are these: First, I estimate the regression weight in each of the replications, and report the mean.
Second, I report the standard deviation of this parameter estimate over the 1000 replications,
which is an empirical estimate of the standard error, and is one way to assess estimation
efficiency. Third, I report the average deviation from the population parameter value to assess
the degree of bias. I also report the "percent relative bias", which is the average deviation,
divided by the empirical standard error, times 100. Collins et al. (2001) used this quantity, and
argued that any value less than about 50 indicated that bias was probably not serious. Finally, I
report the mean squared deviation from the population value, a quantity that is often used to
address efficiency.
Adding Missing Data Variables in FIML/SEM

Simulation 1

Because the manifest variable versions of the three models in question yield identical results, even in the missing data case, only the latent variable versions of these models are involved in the simulations. Each simulation was designed to answer a particular question. First, it has been noted in the literature (e.g., Graham, Hofer, & MacKinnon, 1996; Graham et al., 1997), that FIML/SEM procedures and EM produce identical variance-covariance matrices. It has also been noted that FIML/SEM procedures and EM produce the same regression parameter estimates for regression or path models in which all variables are manifest variables (Graham et al., 1997). For regression or path models based on latent variables, unpublished simulation work has shown that FIML/SEM and EM produce equally unbiased parameter estimates, but that FIML/SEM procedures are very slightly more efficient. The main purpose of Simulation 1 is to replicate these earlier findings, especially the findings from the unpublished simulations.

In Simulation 1, the factor loadings and factor correlations shown in Table 1 were used to generate the data for the population. Note that in this model, there were two latent variables (X and Y), each with four manifest indicators, and an extra variable (RX) which will eventually be used to cause missingness on the indicators of Y, and which is correlated $r = .50$ with the Y latent variable.

In the simulation, data were generated with Genraw (Jöreskog & Hussenius, 1993) based on the covariance matrix implied by these factor loadings and factor covariances (the residual item variances were fixed so that the implied covariance matrix was actually a correlation matrix). N = 1000 cases were generated for each dataset.

Missingness on each of the manifest indicators of the latent variable (Y) was due to RX in the following way:
- if RX=1, probmiss = .05
- if RX=2, probmiss = .35
- if RX=3, probmiss = .65
- if RX=4, probmiss = .95

Missing value status was dealt with separately for each of the four indicators of the latent variable (Y). If a uniformly distributed random variable ("probmiss") had a value greater than or equal to that shown above, the value of all four indicators of Y was not missing. If the value of "probmiss" was less than the value shown, then the value was missing with probability .975. For example, if RX=1, and the variable probmiss was greater than or equal to .05, then all four indicators would have non-missing values. However, if the value of probmiss was less than .05, then each of the indicators had an independent probability of .975 that it would be missing. With this strategy, most cases were either missing or not missing for all four manifest indicators of Y. However, as would be the case with many kinds of real data, a few cases would have partial data for the four manifest indicators of Y.

Results

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It should be noted here that I am referring here to EM procedure that creates a maximum-likelihood variance-covariance matrix (and vector of means). An SEM procedure is then used to analyze that covariance matrix (and vector of means). In other words, I am NOT referring here to an EM procedure that estimates the latent variable regressions directly.
The results of Simulation 1, which appear in Table 2, show that the "spider" model, the "extra DV model" and the Amos analysis of EM covariance matrix were all virtually the same in terms of bias and efficiency.

**Simulation 2**

Although it might have been expected that the models in Simulation 1 would yield similar results when the model was correct, it remains a possibility that the three models would yield different results, either in terms of bias or efficiency, when the model is not correct. Simulation 2 was designed to test this hypothesis.

In this case the data were generated by a model in which there were two large residual covariances, one within the X latent variable, and one within the Y latent variable. These two residual covariances were not modeled in the analysis part of the simulation. RMSEA was just greater than .08 in the population. It seemed reasonable to limit this hypothesis test to models that were at least marginally acceptable. The idea here is that researchers who have a model that might possibly be acceptable, specification of the incorrect model could be a problem with respect to missing data estimation. However, if the model is so bad that it would not be acceptable for other reasons (e.g., bad fit), then the missing data issues would not arise. Thus, we limit this simulation to the situation that is at the edge of acceptability.

Factor loadings were approximately .70 for all items loading on both factors. The RX variable was correlated r=.50 with the latent dependent variable. As with Simulation 1, missingness on the indicators of the latent variable (Y) was due to RX in the following way:

- if RX=1, probmiss = .05
- if RX=2, probmiss = .35
- if RX=3, probmiss = .65
- if RX=4, probmiss = .95

The results, which appear in Table 3, showed that the "spider" model, the "extra DV model" and the Amos analysis of the EM covariance matrix were all about the same in terms of bias and efficiency.

**Simulation 3**

Although the three models performed about equally well when the model was true, and the missingness was MAR-Linear, it could be that the models differ when the missingness is not linear. Simulation 3 tested this hypothesis.

In this case, the analysis model was the same as the model that generated the data. The simulation was set up just like Simulation 1, except that the missingness mechanism was MAR-Convex (see Collins et al., 2001), rather than MAR-linear (see below).

Factor loadings were approximately .70 for all items loading on both factors. The RX variable was correlated r=.50 with the latent dependent variable. Missingness on the latent DV (Y) was due to RX in the following way:

- if RX=1, probmiss = .80
- if RX=2, probmiss = .20
- if RX=3, probmiss = .20
- if RX=4, probmiss = .80

The results of Simulation 3, which appear in Table 4, showed that the "spider" model, the "extra DV model" and the "Amos analysis of EM covariance matrix" were all about the same in terms of bias and efficiency. Although the overall level of bias was slightly higher that in the previous simulations, the overall bias was still only 9% of a standard deviation, which still
Schafer (2000) has suggested that EM and multiple imputation (MI) are not exactly equivalent in that MI produces parameter estimates based on the average of parameter estimates (e.g., b-weights) over the multiple imputations, whereas EM produces parameter estimates on what might be thought of as the average of the covariance matrices. That is, the average of the b-weights may not be precisely the same as the b-weight of the average (covariance matrix).

Simulation 4

Although the three models performed about the same in the first three simulations, the results could have been due to the relatively high saturation of the factor loadings. Simulation 4 tested this hypothesis. Simulation 4 was set up exactly the same as Simulation 1, except that every factor loading was exactly .20 smaller than the corresponding factor loading in Simulation 1. For this simulation, the analysis model was the same as the model that generated the data, and the factor loadings were approximately .50 for all items loading on both factors. The RX variable was correlated r = .50 with the latent dependent variable. Missingness on the manifest indicators of the latent variable (Y) was due to RX in the following way:

- if RX=1, probmiss = .05
- if RX=2, probmiss = .35
- if RX=3, probmiss = .65
- if RX=4, probmiss = .95

The results, which appear in Table 5, show that the "spider" model, the "extra DV model" and the "Amos analysis of EM covariance matrix" were all about the same in terms of bias and efficiency. However, it appeared that the EM estimates were very slightly more biased, and very slightly less efficient than the Extra DV and Spider Models. However, even with 1000 replications, I have low confidence that these differences are reliable, and even if they are reliable, the differences are very small.

Discussion

The simulation results show one thing very clearly: The three approaches to analyzing missing data are approximately equivalent, and all three provide acceptable estimation in the missing data case. If we think of the analysis of the EM covariance matrix as being the "gold standard", then it is clear that either of the two options for FIML/SEM analysis are entirely acceptable alternatives. It is true that EM and multiple imputation are not exactly equivalent, but the two are known to yield highly similar estimates, assuming a sufficiently large number of imputations. Thus, to the extent that EM and MI yield equivalent results, the results of the present study confirm that MI and FIML/SEM procedures can, in theory, be equally unbiased and efficient in a wide variety of missing data contexts.

The results show that the "Extra DV" and "Spider" models work as well as EM when the analysis model is that same as the model that generated the data. The results also show that the "Extra DV" and "Spider" models work as well as EM when the analysis model is almost, but not quite unacceptable in terms of fit (of course, as we note below, this was but one example of a model with bad fit). Note that this latter comparison was with the model actually tested, not the true model. The question here was whether the "badness" of the model would affect the missing data estimation. Clearly, at least in this case, the answer is "no".

The results also show that the "Extra DV" and "Spider" models each perform as well as

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2 Schafer (2000) has suggested that EM and multiple imputation (MI) are not exactly equivalent in that MI produces parameter estimates based on the average of parameter estimates (e.g., b-weights) over the multiple imputations, whereas EM produces parameter estimates on what might be thought of as the average of the covariance matrices. That is, the average of the b-weights may not be precisely the same as the b-weight of the average (covariance matrix).
EM when the mechanism of missingness, although MAR, was not linear. Finally, the results show that the "Extra DV" and "Spider" models each performed as well as, and perhaps even very slightly better than EM, when the saturation for the factor loadings was low (loadings approximately .50 rather than .70). This latter test was performed when the analysis model was the same as the model that generated the data.

**Limitations of the Study**

In this study, several possible sets of circumstances were tested that might have produced noticeable differences between EM on the one hand, and the two FIML/SEM models on the other. I was also looking for circumstances that might lead to differences in bias and/or efficiency between the two FIML/SEM models. The fact that no such circumstances were found is a bit like trying to accept the null hypothesis. It is difficult to conclude from these four simulations that these three models are essentially the same across a wide variety of circumstances.

Nonetheless, there are many reasons to believe that the three model will perform similarly for bias and efficiency in a wide variety of models and missing data contexts. First, it is known that the manifest-variable versions of these three models are not just equivalent, but yield identical parameter estimates in the missing data case (Graham et al., 1994; 1996). Second, the evidence from the simulations of this study provide some confidence in the rough equivalence of the three models.

It could be argued that some of the simulation conditions were not particularly extreme, and that more extreme conditions would produce differences between and among the three models. However, consider this: The relatively bad fitting model (Simulation 2) was about as extreme as it could be before most researchers would reject the model for reasons irrelevant to missing data. That is, the fact that most researchers would reject such poor-fitting models anyway, they are unlikely to make mistakes relating to the missing data aspects of the model.

Similarly, it could be argued that the simulation addressing the magnitude of factor loadings (Simulation 4) could have been more extreme. On the other hand, with loadings much lower than .50, there is likely to be considerable empirical underidentification. Thus, again, the researcher may reject the model before the issue proper handling of missing data even comes up.

**Practicalities of Adding (Irrelevant) Variables to the SEM Model**

The results of this article show that, in theory, FIML/SEM can be as good for a wide variety of missing data situations as multiple imputation. However, in multiple imputation, the solution is very easy to implement, and, in fact, is pretty much how researchers use multiple imputation all the time. With FIML/SEM, however, this is not how most researchers approach their models when they have missing data. As Collins et al. (2001) have pointed out, FIML/SEM users are somewhat discouraged by the program to add extraneous variables to the model.

The "Extra DV" and "Spider" models both are quite easy to implement in the graphics version of Amos when the model itself is on the small side, and when there are only a couple extra variables to add. However, when the model of interest begins to get even moderately large (e.g., three latent independent variables, two latent mediating variables, and three latent dependent variables), and the number of extra variables gets as large even as five, the graphics version of Amos begins to be overloaded.

On the other hand, it is relatively straightforward to add extraneous variables in the text version of Amos, regardless of the size of the model. Still, it would be nice if an option existed in the graphics versions of these FIML/SEM programs, that would allow the researcher to
include extraneous variables in convenient way, without cluttering up the model of substantive interest.

References


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Model for Generating Data

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Figure 1
Model 1: "EM Model" (Manifest)
Figure 2
"EM Model" (Latent)
Figure 3
Model 2: "Extra DV" Model (Manifest)
Figure 4
Model 2: "Extra DV" Model (Latent)
Figure 5
Model 3: "Spider" Model (Manifest)
Figure 6
Model 3: "Spider" Model (Latent)